

浙江大学 2018-2019 学年 秋冬 学期

《离散数学》课程期末考试试卷

课程号: 21120401 开课学院: 计算机学院

考试试卷: ☒ A卷 ☐ B卷

考试形式: ☒ 闭卷 ☐ 开卷, 允许带 _____ 入场

考试日期: 2019 年 1 月 24 日, 考试时间: 120 分钟

诚信考试, 沉着应考, 杜绝违纪

考生姓名 _____ 学号 _____ 所属院系 _____

| 题序 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 总分 |
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| 得分 | | | | | | | | | |
| 评卷人 | | | | | | | | | |

ZHEJIANG UNIVERSITY
DISCRETE MATHEMATICS, FALL-WINTER 2018
FINAL EXAM

1. (20 pts) Determine whether the following statements are true or false. If it is true fill a \checkmark otherwise a \times in the bracket before the statement.

- (a) () There are two sets A and B , such that both $A \subseteq B$ and $A \in B$.
- (b) () Let A, B be two sets. If $2^A \subseteq 2^B$, then $A \subseteq B$, where 2^X is the power set of X .
- (c) () Let φ be a proposition with three variables p, q , and r , if exactly one of the assignments 000, 011, 100, and 111 make φ true, then φ can be converted in full disjunctive normal form $\Sigma(1, 2, 5, 6)$.
- (d) () Let $P = \{f | f \text{ is a propositional formula with variables } p, q, r\}$ and R be the equivalent relation on P by $\varphi R \psi$ iff $\varphi \Leftrightarrow \psi$, for $\varphi, \psi \in P$, then the number of equivalence classes of P on R is 256.
- (e) () Let $P(x), Q(x)$ be two predicates, then $\forall x P(x) \rightarrow \exists y Q(y) \Leftrightarrow \forall x \exists y (P(x) \rightarrow Q(y))$.
- (f) () There is a binary relation R on set A such that R is both an equivalence relation and partial order on A .
- (g) () Suppose (A, \preceq) is a finite nonempty poset. Then A has a minimal element.
- (h) () There is no known algorithm to decide if a graph has a Hamilton circuit.
- (i) () Every connected bipartite graph contains a circuit of even length.
- (j) () A binary tree with height h has at most 2^h leaves.

2. **(12 pts)** Construct a logical argument using rules of inference to show that the following sentences imply the conclusion: *It rained.*

If it does not rain or if it is not foggy, then the sailing race will be held and the life-saving demonstration will go on. If the sailing race is held, then the trophy will be awarded. The trophy was not awarded.

Justify each step by indicating the rule you applied.

3. **(12%)** Prove that the two sets of real numbers $(0, 1)$ and $[a, b) = \{x \in \mathbb{R} | a \leq x < b\}$ have the same cardinality(基数).

4. **(10 pts)** Let $S \subseteq \{1, 2, 3, 4, 5, 6, \dots, 98, 99, 100\}$ where $|S| = 62$. Prove that there exist $x, y \in S$ such that $x - y = 23$.

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5. **(12 pts)** Define a binary relation \mathcal{R} on $\mathbb{R} \times \mathbb{R}$ by $(x, y)\mathcal{R}(a, b)$ if and only if $y - b = 5(x - a)$.
- (a) Show that \mathcal{R} is an equivalence relation on $\mathbb{R} \times \mathbb{R}$.
 - (b) Find the equivalence classes of $(0, 0)$ and $(1, -2)$.
 - (c) Describe the geometric meaning the equivalence class $[(a, b)]$.
6. **(12 pts)** Let $G = (V, E)$ be a simple graph with n vertices, where $n \geq 3$, and $d(u) + d(v) \geq n$, for every $u, v \in V$. Prove
- (a) G is connected.
 - (b) G is a Hamilton graph.

7. **(10 pts)** Find all non-isomorphism trees with exactly 6 vertices and four or more leaves.

8. **(12 pts)**

(a) Let a_n denote the number of integer solutions to

$$5x_1 + 4x_2 + x_3 + x_4 = n, 0 \leq x_1, 0 \leq x_2, 0 \leq x_3 \leq 4, 0 \leq x_4 \leq 3$$

(i) Find the generating function $G(x) = \sum_{n=0}^{\infty} a_n x^n$. (ii) Obtain a formula for a_n , explicitly.

- (b) For all integers n , let b_n be the number of binary strings of length n that contain the substring 000. (i) Give b_4, b_5, b_6 . (ii) Write a recurrence relation for b_n . (iii) Solve the recurrence relation for b_n .