# 浙江大学 2020-2021 学年 秋冬 学期

## 《离散数学》课程期末考试试卷

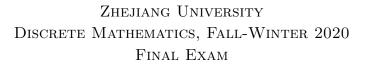
课程号: <u>21120401</u> 开课学院: <u>计算机学院</u>
考试试卷: ☑ A卷 □ B卷
考试形式: ☑ 闭卷 □ 开卷,允许带 \_\_\_\_\_入场
考试日期: <u>2021</u> 年<u>1</u>月<u>26</u>日,考试时间: <u>120</u>分钟

#### 诚信考试,沉着应考,杜绝违纪

考生姓名\_\_

学号\_\_\_\_\_\_所属院系\_

题序	1	2	3	4	5	6	7	总分
得分								
评卷人								



- 1. (20 pts) Determine whether the following statements are true or false. If it is true fill a  $\sqrt{}$  otherwise a  $\times$  in the bracket before the statement.
  - (a) ( ) Let A, B and C be arbitrary sets. If  $A C \subseteq B C$ , then  $A \cup C \subseteq B \cup C$ .
  - (b) ( ) Let A, B be two sets. If  $\rho(A) \subseteq \rho(B)$ , then  $A \subseteq B$ , where  $\rho(X)$  is the power set of X.
  - (c) ( ) Let P(x) be a predicate, then  $\forall x P(x) \to Q \Leftrightarrow \forall x (P(x) \to Q)$ , where Q is independent of x.
  - (d) ( ) The poset ( $\{1, 2, 4, 8, 12, 16, 32\}$ , |) is a lattice(格), where  $x \mid y$  denote x divides y.
  - (e) ( ) Let  $(S, \preceq)$  be a partially ordered set, if there is unique maximal element a of S, then a is the greatest element of S.
  - (f) ( ) If the following assignments 000,011 and 110 make the propositional formula  $\varphi$  false, then  $\varphi$  can be converted in full conjunctive normal form  $\Pi(0,3,6)$ .
  - (g) ( ) The set of all functions from  $\mathbb{N}$  to  $\{0,1\}$  is countably infinite.
  - (h) ( ) If there are 800 people in a room then at least 3 of them are guaranteed to have the same birthday.
  - (i) ( ) All simple complete graphs with at least 3 vertices are Euler graphs.
  - (j) ( ) In a binary tree with n vertices and l leaves, then  $2 \cdot l \leq n+1$ .

2. (12 pts) ON MATHEMATICAL LOGIC

Construct arguments to prove that the following reasoning is valid.

HYPOTHESIS:  $\neg p \lor q \to r, s \lor \neg q, \neg t, p \to t, \neg p \land r \to \neg s$ Conclusion:  $\neg q$ 

3. (10 pts) On Infinite sets

Let A be an arbitrary infinite set, B be a countably infinite set, and  $A \cap B = \emptyset$ . Prove that sets A and  $A \cup B$  have the same cardinality.

#### 4. (12 pts) ON GRAPH

Let G be a simple graph with n vertices and k connected components.

- (a) What is the minimum possible number of edges of G?
- (b) What is the maximum possible number of edges of G?

5. (24 pts) ON SET AND RELATION

Let A be a set with n elements and  $B = \{a, b, c\}$ .

- (a) How many different symmetric relations on A?
- (b) How many different anti-symmetric relations on A?
- (c) How many both symmetric and antisymmetric binary relations on A are there?
- (d) How many different equivalence relations are there on B?
- (e) How many different partial order relations are there on B?
- (f) Is there a binary relation R on B such that R is both an equivalence relation and a partial order? Either give an example, or show that no such R exist.

Justify your answer, but you don't need to give a formal proof.

#### 6. (10 pts) ON TREE

Suppose that T is a tree of 9 vertices with a vertex of 6 degrees.

- (a) What degree sequences can T have?
- (b) Draw all non-isomorphic trees of 9 vertices with a vertex of 6 degrees.

### 7. (12 pts) ON COUNTING

Let  $b_n$  denote the number of binary strings of length n that contain 101 as a substring and  $B(x) = \sum_{n=1}^{\infty} b_n x^n$ .

- (a) Determine the value of  $b_1, b_2, b_3, b_4, b_5$ .
- (b) Derive an explicit closed-form expression for B(x).

HINT: You might want to set up recurrence relation for the appropriate sequences.