浙江大学 2020-2021 学年 秋冬 学期

《离散数学》课程期末考试试卷

课程号：21120401 开课学院：计算机学院

考试试卷：☑ A卷 ☐ B卷

考试形式：☑ 闭卷 ☐ 开卷，允许带 ________入场

考试日期: 2021 年 1 月 26 日，考试时间: 120 分钟

诚信考试, 沉着应考, 杜绝违纪

考生姓名________________ 学号________________ 所属院系________________

题序	1	2	3	4	5	6	7	总分
得分								
评卷人								

Zhejiang University

Discrete Mathematics, Fall-Winter 2020

Final Exam

1. **(20 pts) Determine whether the following statements are true or false. If it is true fill a $\surd$ otherwise a $\times$ in the bracket before the statement.**

   (a) ( ) Let A, B and C be arbitrary sets. If $A-C \subseteq B-C$, then $A \cup C \subseteq B \cup C$.

   (b) ( ) Let A, B be two sets. If $\rho(A) \subseteq \rho(B)$, then $A \subseteq B$, where $\rho(X)$ is the power set of X.

   (c) ( ) Let $P(x)$ be a predicate, then $\forall x P(x) \to Q \Leftrightarrow \forall x(P(x) \to Q)$, where Q is independent of x.

   (d) ( ) The poset $(\{1,2,4,8,12,16,32\}, |)$ is a lattice(格), where $x \mid y$ denote x divides y.

   (e) ( ) Let $(S, \preceq)$ be a partially ordered set, if there is unique maximal element a of S, then a is the greatest element of S.

   (f) ( ) If the following assignments $000, 011$ and 110 make the propositional formula φ false, then φ can be converted in full conjunctive normal form $\Pi(0,3,6)$.

   (g) ( ) The set of all functions from $\mathbb{N}$ to $\{0,1\}$ is countably infinite.

   (h) ( ) If there are 800 people in a room then at least 3 of them are guaranteed to have the same birthday.

   (i) ( ) All simple complete graphs with at least 3 vertices are Euler graphs.

   (j) ( ) In a binary tree with n vertices and l leaves, then $2 \cdot l \leq n+1$.

2. **(12 pts)** ON MATHEMATICAL LOGIC
Construct arguments to prove that the following reasoning is valid.

HYPOTHESIS: $\neg p \vee q \to r$, $s \vee \neg q$, $\neg t$, $p \to t$, $\neg p \wedge r \to \neg s$
CONCLUSION: $\neg q$

3. **(10 pts)** ON INFINITE SETS
Let A be an arbitrary infinite set, B be a countably infinite set, and $A \cap B = \emptyset$. Prove that sets A and $A \cup B$ have the same cardinality.

4. **(12 pts)** ON GRAPH
Let G be a simple graph with n vertices and k connected components.

(a) What is the minimum possible number of edges of G?

(b) What is the maximum possible number of edges of G?

5. **(24 pts)** ON SET AND RELATION
   Let A be a set with n elements and $B = \{a, b, c\}$.
   (a) How many different symmetric relations on A?
   (b) How many different anti-symmetric relations on A?
   (c) How many both symmetric and antisymmetric binary relations on A are there?
   (d) How many different equivalence relations are there on B ?
   (e) How many different partial order relations are there on B?
   (f) Is there a binary relation R on B such that R is both an equivalence relation and a partial order? Either give an example, or show that no such R exist.

   Justify your answer, but you don't need to give a formal proof.

6. **(10 pts)** ON TREE

   Suppose that T is a tree of 9 vertices with a vertex of 6 degrees.
   (a) What degree sequences can T have?
   (b) Draw all non-isomorphic trees of 9 vertices with a vertex of 6 degrees.

7. **(12 pts)** ON COUNTING

Let b_n denote the number of binary strings of length n that contain 101 as a substring and $B(x) = \sum_{n=1}^{\infty} b_n x^n$.

(a) Determine the value of b_1, b_2, b_3, b_4, b_5.

(b) Derive an explicit closed-form expression for $B(x)$.

HINT: You might want to set up recurrence relation for the appropriate sequences.

ENJOY YOUR SPRING FESTIVAL!