Constants: $e = 1.6\times10^{-19}$ C, $\varepsilon_0 = 8.85\times10^{-12}$ C²/N m², $m_e = 9.11\times10^{-31}$ kg, $\mu_0 = 4\pi\times10^{-7}$ T m/A, $h = 6.63\times10^{-34}$ J s,

I. Fill in the space underlined. (50% in total)

$i = nesv$
$j = nev$

1. Near the Earth, the density of protons (质子的密度) in the solar wind (太阳风) is **8.70 cm⁻³**, and their speed is **470 km/s**. The current density of these protons is $6.5424\times10^{-7}\ A/m^2$. If the Earth's magnetic field did not deflect (偏转) them, the protons would strike the Earth. The total current received by the Earth would be $6.5424\times10^{-7}\times 4\pi R^2$ (R为地球半径)

2. A dielectric slab of thickness b (厚度为b的电介质板) is inserted between the plates of parallel-plate capacitor (平行板电容器) of plate separation d and area A. The capacitance (电容) is given by $\cancel{\frac{A\varepsilon_0}{d-b}}$ $\frac{A\varepsilon_0}{k_e d-(k_e-1)b}$

3. As shown in Fig. 1, a thin plastic disk (塑料盘) of radius R has a charge q uniformly distributed (均匀分布) over its surface. If the disk rotates at an angular frequency (角频率) ω about its axis, the magnetic field B at the center (point O) of the disk is of $\frac{\mu_0 \omega q}{2\pi R}$.

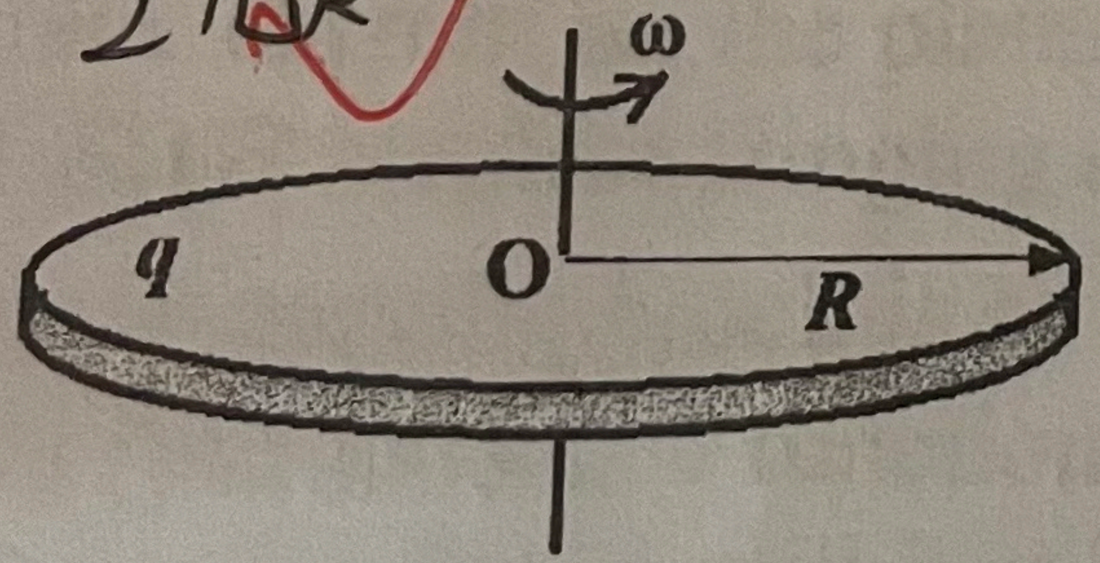


Figure 1

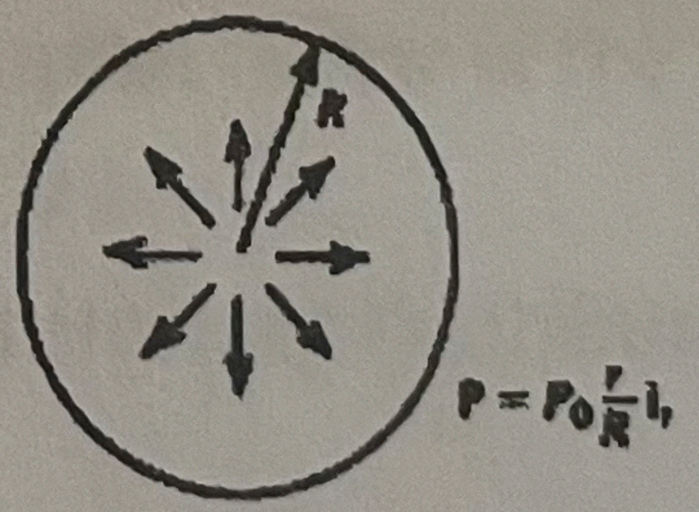


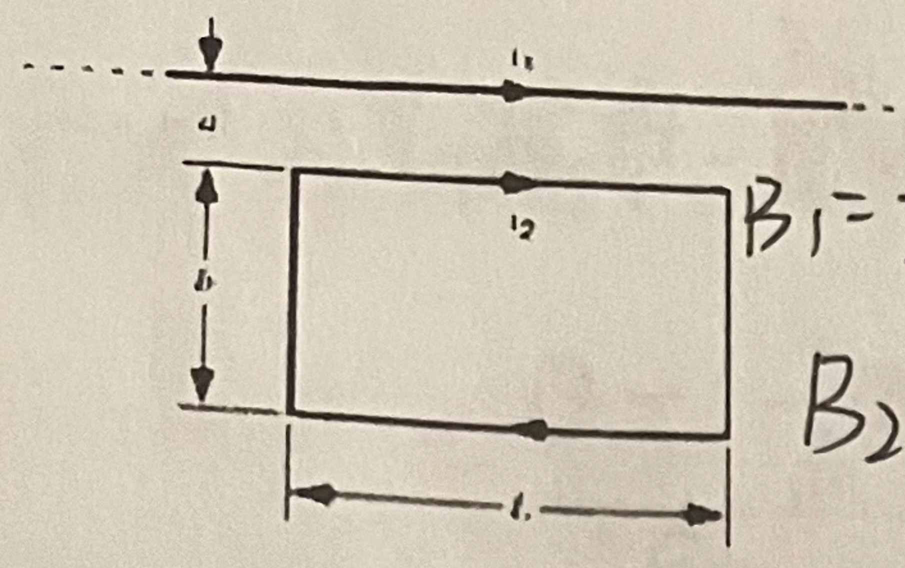
Figure 2

4. As shown in Fig. 2, there is a permanently polarized insulating sphere (永久极化的绝缘球) of radius R with the polarization (极化强度) $\vec{P} = P_0 \frac{r}{R}\hat{r}$

The electric field E_{in} = $-\frac{3P_0 r}{R\varepsilon_0}$ inside sphere, and E_{out} = 0 outside sphere, respectively.

5. Figure 3 shows a long wire carrying a current i_1. The rectangular (长方形) loop carries a current i_2. The resultant force acting on the loop is of $\underline{3.27\times10^{-3}\,N}$. Assume that a = 1.10 cm, b = 9.20 cm, L = 32.3 cm, i_1 = 28.6 A, and i_2 = 21.8 A.

$\Delta B \cdot I_2 L$



$B_1 = \frac{\mu_0 i}{2\pi a} =$

$B_2 = \frac{\mu_0 i}{2\pi(a+b)} -$

Figure 3

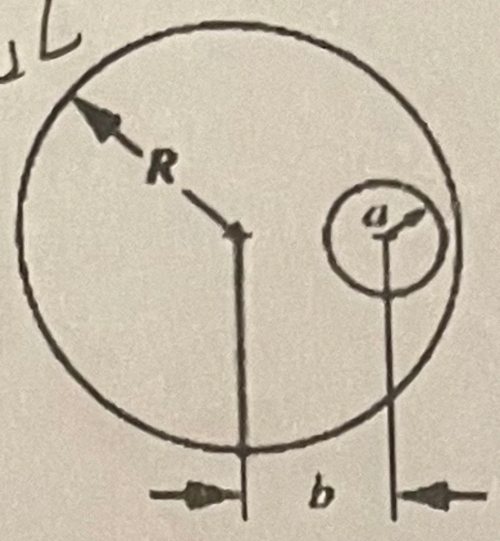


Figure 4

6. Figure 4 shows a cross section of a long, cylindrical conductor (圆柱形导体) of radius R containing a long, cylindrical hole of radius a. The axes of the two cylinders are parallel and are a distance b apart. A current i is uniformly distributed (均匀分布) over shaded area in the figure. The magnetic field at the center of the hole is of $\underline{\frac{\mu_0 i(2\pi Rb-a^2)}{2\pi b(R^2-a^2)}}$.

7. A uniform magnetic field B is changing in magnitude at a constant rate dB/dt. You are given a mass m of copper (铜) with a resistivity (电阻率) ρ and a density (密度) δ, that is to be drawn into a wire (拉成线) and formed into a circular loop (圆环). The induced current (感应电流) in the loop is given by $\underline{\frac{m}{4\pi\rho\delta}\times\frac{dB}{dt}}$, assuming B perpendicular to the loop.

$V = \frac{m}{\delta} = L\cdot S$

8. As shown in Fig. 5, the spherical (球壳) region $a < r < b$ carries a charge per unit volume (体密度) of $\rho = A/r$, where A is a constant. There is a point charge q at the center ($r = 0$) of the enclosed cavity. When A = $\underline{\frac{q}{2\pi a^2}}$, so that the electric field in the region $a < r < b$ has constant magnitude (常数值)?

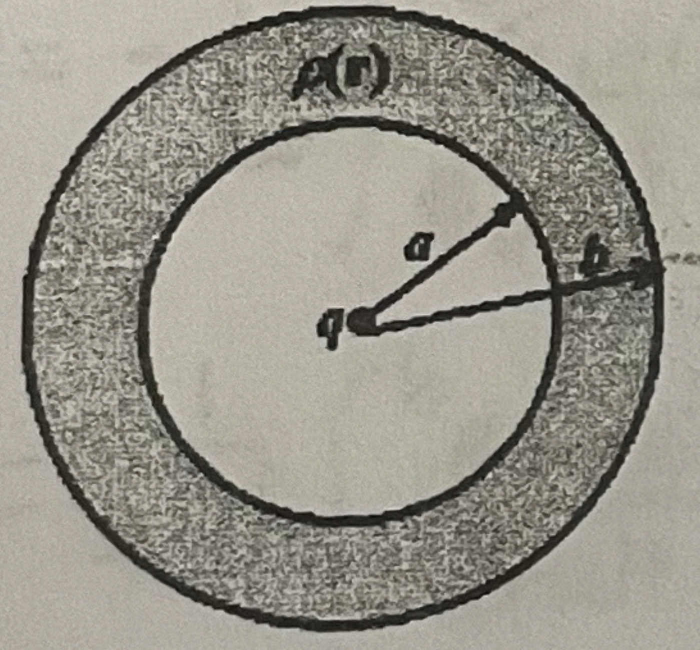


Figure 5

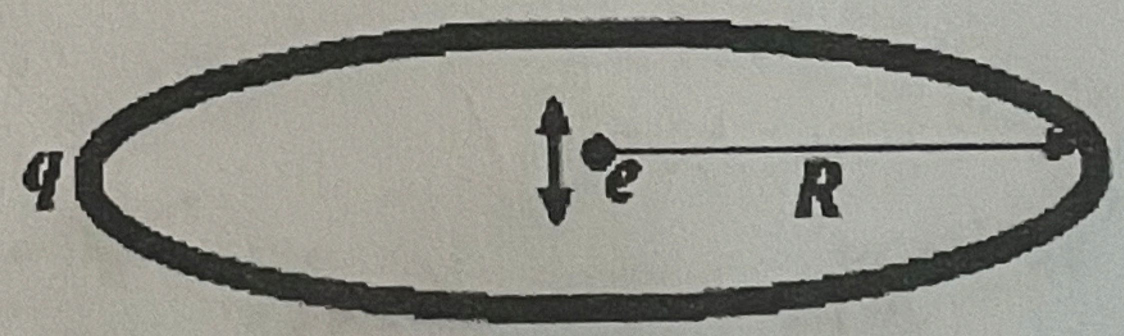


Figure 6

$F = \frac{1}{4\pi\varepsilon_0}\frac{zqe}{(R^2+z^2)^{\frac{3}{2}}} = -kz$

9. As shown in Fig. 6, an electron is constrained to move along the axis of the ring (环) with a charge q. If the electron can perform small oscillations (微小振动) through the center of the ring, its oscillation frequency is given by $\underline{\sqrt{\frac{eq}{16\pi^3\varepsilon_0 R^3 m_e}}}$.

$F = -\frac{1}{4\pi\varepsilon_0}\frac{zqe}{R^3} = -kz$

$k = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qe}{R^3}$

10. The electric field inside a nonconducting sphere (非导体球) of radius R, containing uniform charge density, is radially (径向) directed and has magnitude

$$E = \frac{qr}{4\pi\varepsilon_0 R^3}$$

where q is the total charge in the sphere and r is the distance from the center of the sphere. The potential V = $\underline{-\frac{qr^2}{8\pi\varepsilon_0 R^3}}$ inside the sphere, taking $V = 0$ at $r = 0$. If we take $V = 0$ at $r = \infty$, then the potential V = $\underline{\frac{3qR^2-qr^2}{8\pi\varepsilon_0 R^3}}$.

II. Problems (Present the necessary equations in solution) (50%)

1. **(10%)** If we assume that an atom is composed of a nuclear (原子核) with a charge Q and an electron cloud (电子云) with a charge $-Q$, which distributes uniformly (均匀分布) in a sphere with radius R, as shown in Fig.7. Please estimate the binding energy (结合能) of the atom.

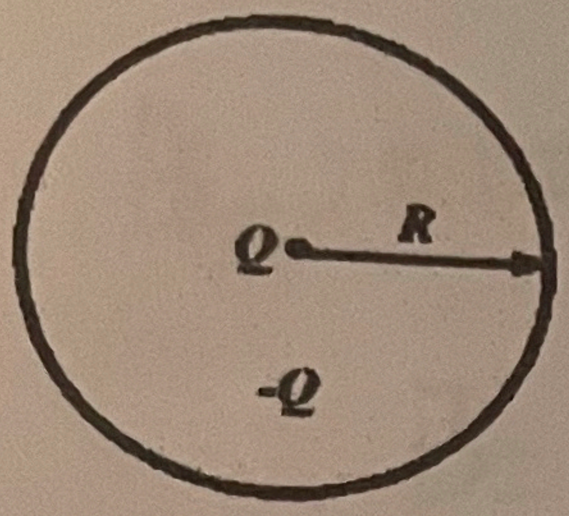


Figure 7

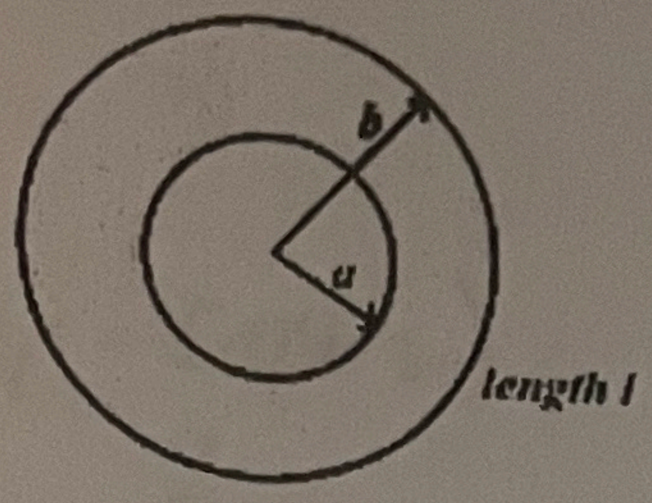


Figure 8

2. **(10%)** As shown in Fig. 8, coaxial cylindrical electrodes (同轴圆柱形电极) of length l with respective radii a and b enclose an Ohmic material, whose conductivity (电导率) varies linearly with radius from σ_1 at the inner cylinder to σ_2 at the outer, as

$$\sigma = \sigma_1 + (\sigma_2 - \sigma_1)(\frac{r-a}{b-a})$$

Please calculate its resistance (电阻).

3. **(15%)** As shown in Fig. 9, there is a coaxial cable (同轴电缆) made of superconducting material (超导材料，$\sigma\to\infty$), and having short circuited end (短路端) free to move (可自由运动) along the x axis.
   (a) What is the inductance (自感系数) of the cable as a function of x?
   (b) What is the force on the end?

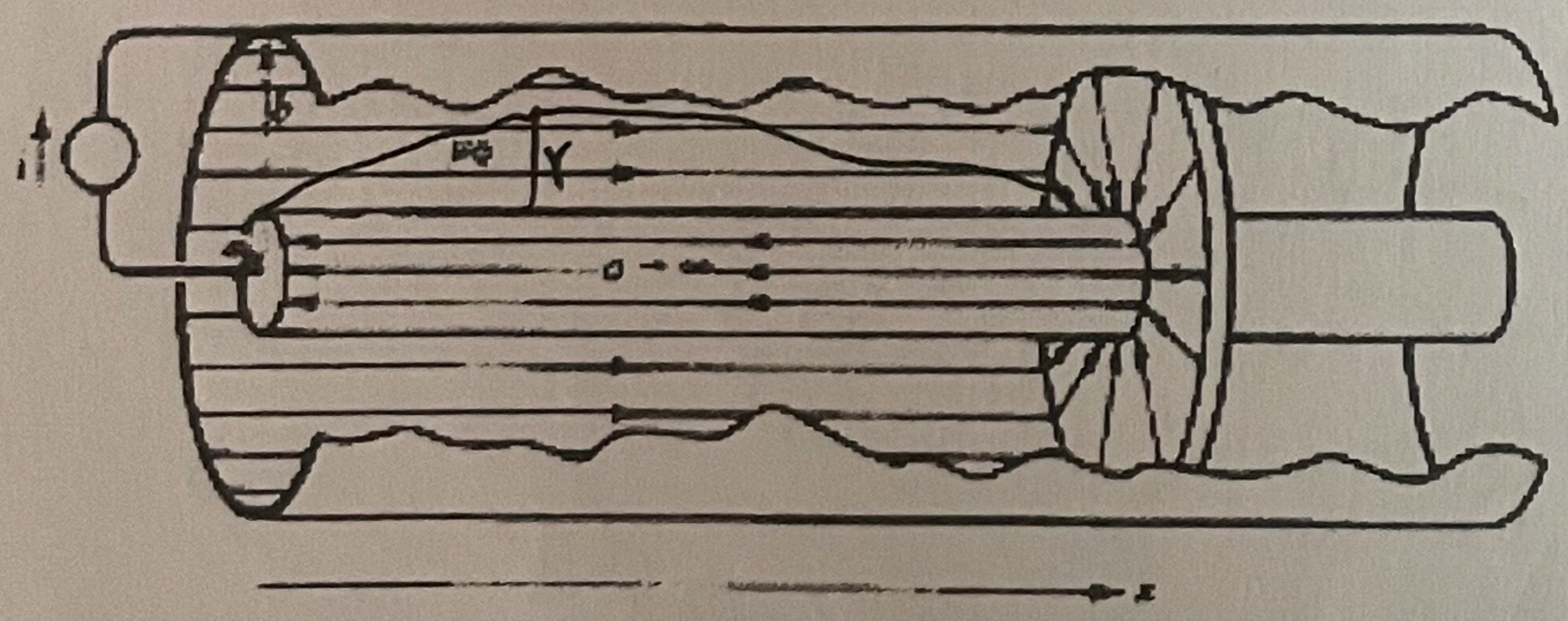


Figure 9

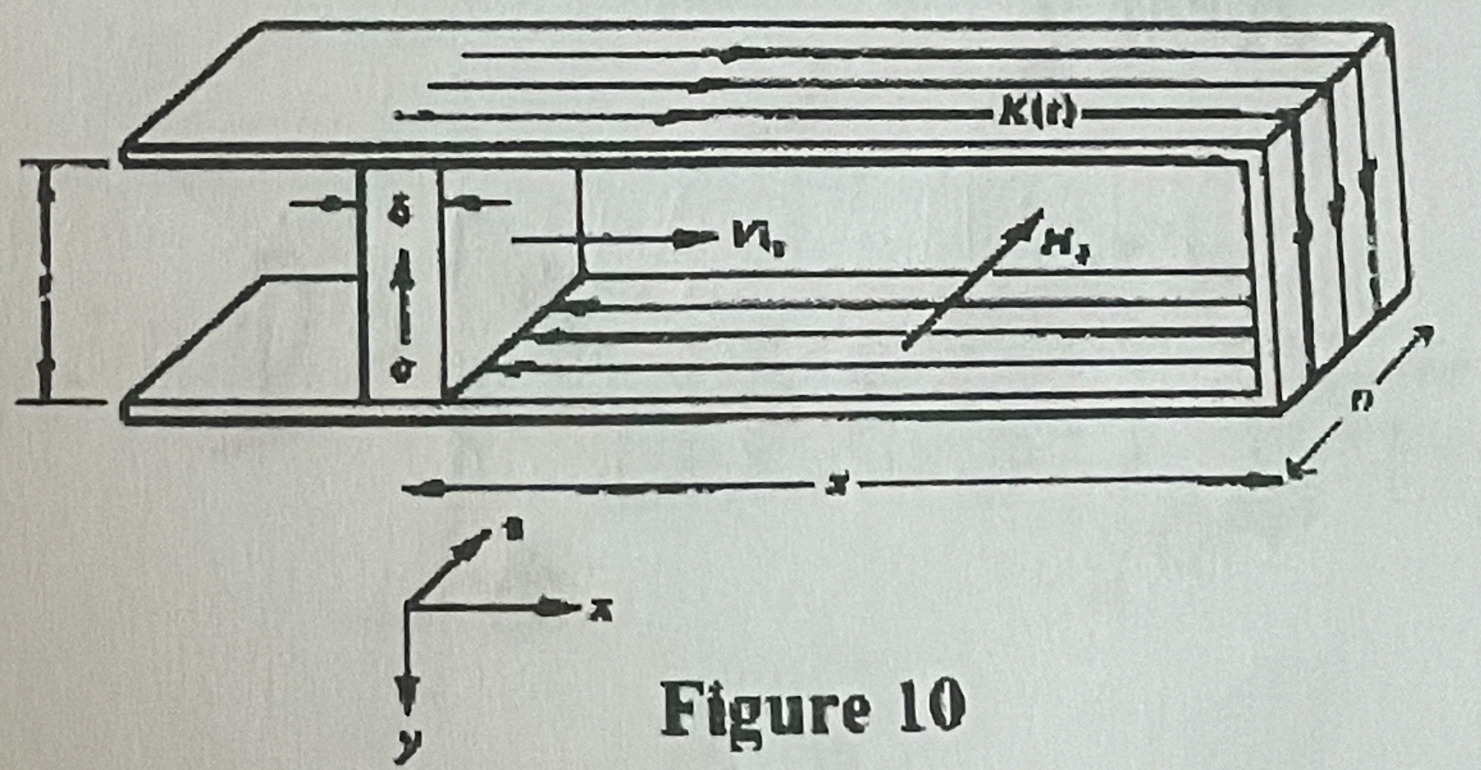


Figure 10

4. **(15%)** As shown in Fig. 10, a thin block (薄的金属块) with conductivity σ and thickness δ moves with constant velocity $v i_x$ between short circuited (短路) superconducting parallel plates (超导平行板). An initial surface current (起始表面电流) K_0 (the current per width) is imposed at $t = 0$ when $x = x_0$, but the source is then removed.
   (a). The surface current on the plates $K(t)$ will vary with time. What is the magnetic field in term of $K(t)$? Neglect fringing effects (忽略边缘效应).
   (b). Because the moving block is so thin, the current is uniformly distributed over the thickness δ. Please find $K(t)$ as a function of time.
   (c). What value of velocity will just keep the magnetic field constant with time until the moving block reaches the end?